CHAPTER 8

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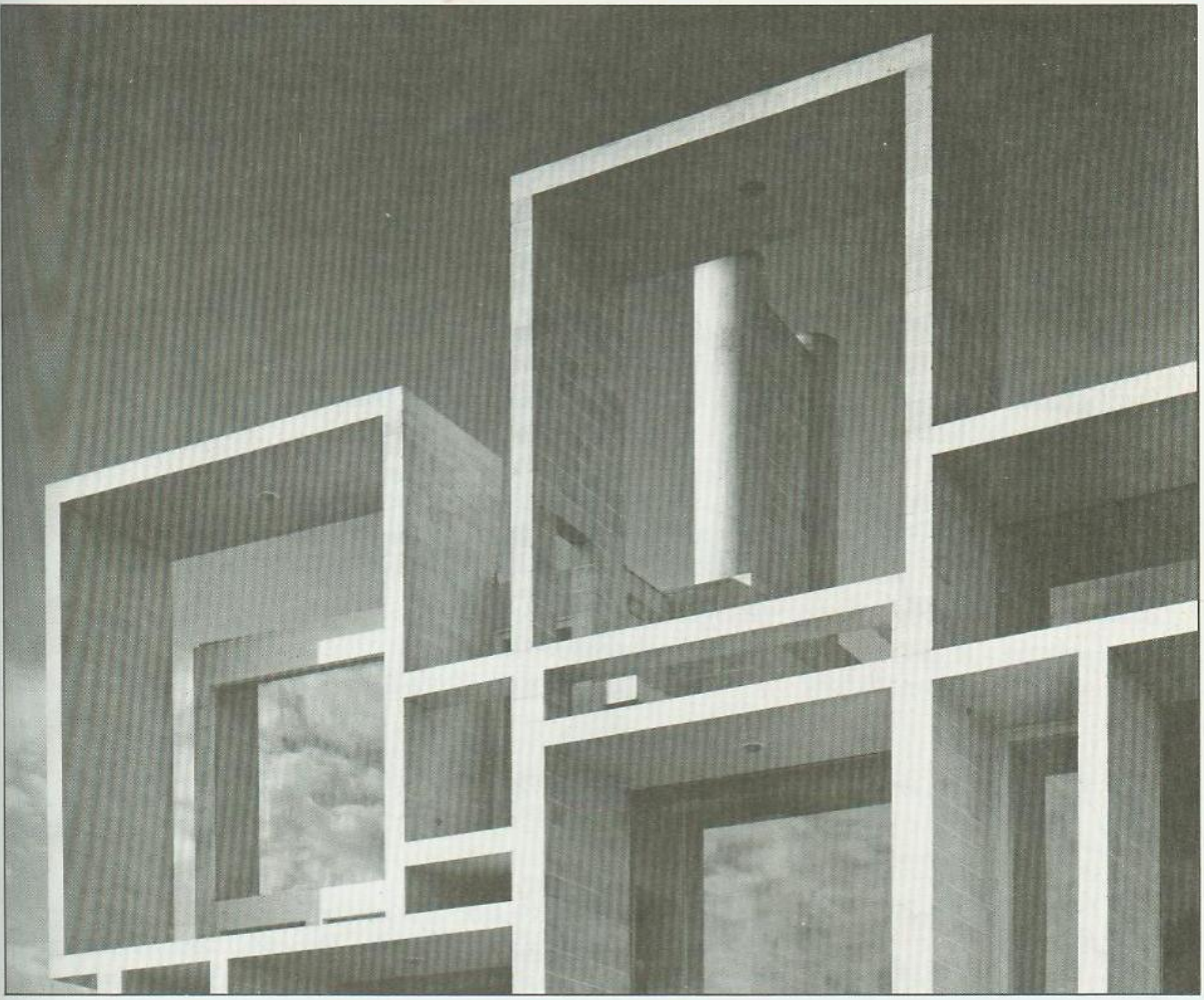
Quadrilaterals and Polygons

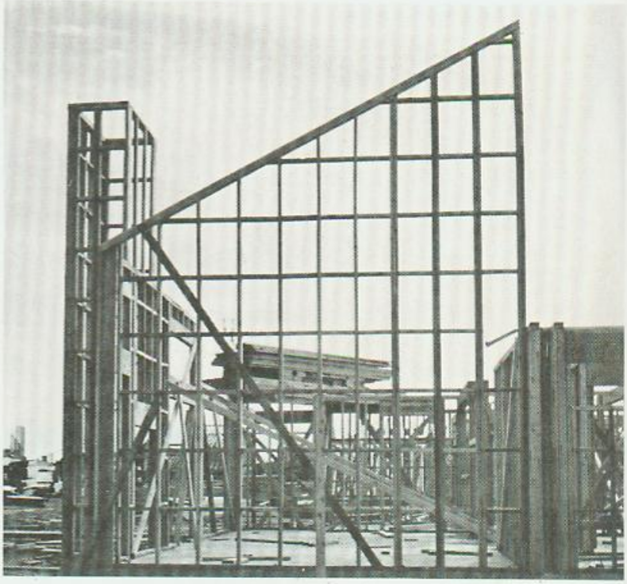


8–1 Quadrilaterals

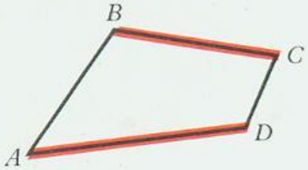
REVIEW: A **quadrilateral** is the union of four segments determined by four points, no three of which are collinear. The segments intersect only at the endpoints.

Our world is full of examples of four-sided figures of all shapes and sizes. We can classify them in terms of sides, angles, and relationships between sides and angles.

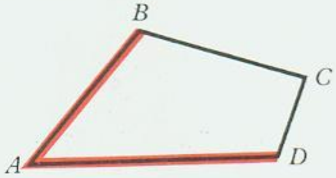
In this chapter we shall study these classifications and learn some properties of quadrilaterals.



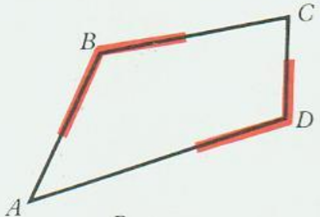
The figures below illustrate some important terms for quadrilaterals.



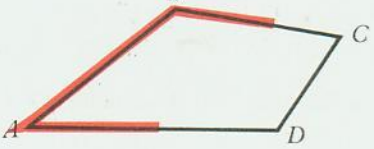
Sides $\overline{BC}$ and $\overline{AD}$ have no vertex in common. They are a pair of *opposite sides.* Sides $\overline{AB}$ and $\overline{DC}$ are also opposite sides.



Sides $\overline{AB}$ and $\overline{AD}$ have a vertex in common. They are a pair of *adjacent sides.* Other pairs of adjacent sides are $\overline{AB}$ and $\overline{BC}$, $\overline{BC}$ and $\overline{CD}$, and $\overline{AD}$ and $\overline{DC}$.

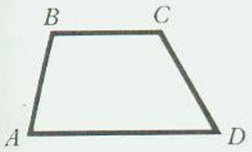


Angles B and D have no side in common. They are a pair of *opposite angles.* Angles A and C are also opposite angles.



Angles A and B have side $\overline{AB}$ in common. They are a pair of *adjacent angles.* Other pairs of adjacent angles are $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, and $\angle D$ and $\angle A$.

We now describe the basic types of quadrilaterals.

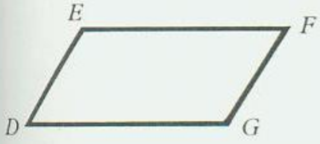


One pair of opposite sides are parallel.

ABCD is a *trapezoid.* $\overline{BC}$ and $\overline{AD}$ are *bases* of the trapezoid.

Definition 8-1

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides.

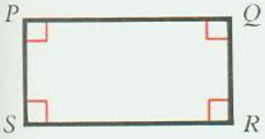


Both pairs of opposite sides are parallel.

EDGF is a *parallelogram.*

Definition 8-2

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

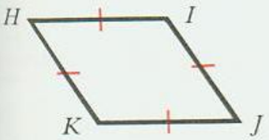


All four angles are right angles.

PQRS is a *rectangle* (and also a parallelogram).

Definition 8-3

A **rectangle** is a parallelogram with four right angles.

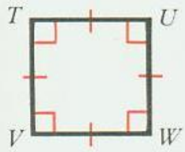


All four sides are congruent.

HIJK is a *rhombus* (and also a parallelogram).

Definition 8-4

A **rhombus** is a parallelogram with four congruent sides.



All four angles are right angles. All four sides are congruent.

TUWV is a *square* (and also a rectangle, a rhombus, and a parallelogram).

Definition 8-5

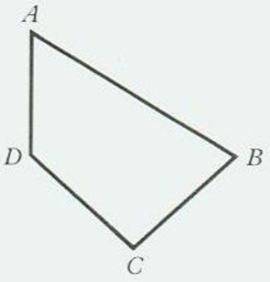
A **square** is a rectangle with four congruent sides.

EXERCISES

A.

For exercises 1–4 refer to quadrilateral *ABCD*.

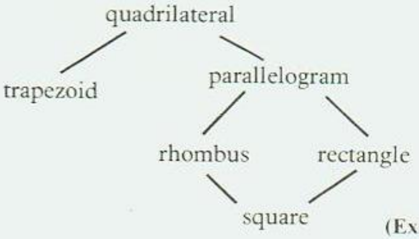
1. Name the side opposite $\overline{AB}$.
2. Name the angles adjacent to $\angle C$.
3. Name the sides adjacent to $\overline{BC}$.
4. Name the angle opposite $\angle D$.



(Exs. 1- 4)

Answer true or false for exercises 5–12.

5. A square is a rectangle.
6. A rectangle is a parallelogram.
7. A parallelogram is a rhombus.
8. A trapezoid is a parallelogram.
9. Some parallelograms are rectangles.
10. A rhombus is a square.
11. Some rhombuses are rectangles.
12. A parallelogram is a trapezoid.



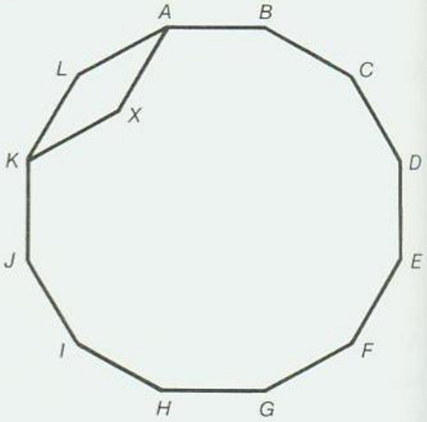
(Exs. 5–12)

B.

13. Construct a parallelogram with a 30° angle.
14. Construct a rhombus with a 60° angle.
15. Construct a trapezoid with two right angles.
16. Construct a rectangle with its width one half its length.

Activity

1. Trace this regular 12-gon and the parallelogram with sides $\overline{AL}$ and $\overline{LK}$.
2. Draw a parallelogram with sides $\overline{KJ}$ and $\overline{KX}$. Draw another with sides $\overline{AX}$ and $\overline{AB}$.
3. Guess how many parallelograms result if this process continues until the 12-gon has been divided into parallelograms.
4. Complete your construction to check your guess.



17. Construct a trapezoid with a pair of 45° base angles.

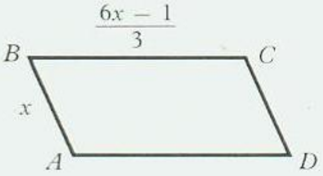
Draw and name the quadrilateral described in each of exercises 18–20.

18. The quadrilateral has two pairs of parallel sides, no right angles, and no pair of adjacent sides congruent.

19. The quadrilateral has at least one pair of congruent adjacent sides, at least one pair of congruent opposite sides, and no right angles.

20. The quadrilateral has at least one pair of parallel sides, no congruent adjacent sides, and exactly one pair of congruent opposite sides.

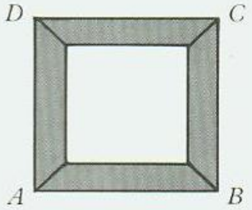
C.



(Ex. 21)

21. The perimeter (distance around) the parallelogram is 32 cm. What is the length of each side (to the nearest millimeter)?

22. The longer base of a trapezoid is the square of the shorter base. The nonparallel sides are congruent. The nonparallel side is 3 more than the shorter base. If the perimeter of the trapezoid is 24 cm, what are the lengths of the sides?

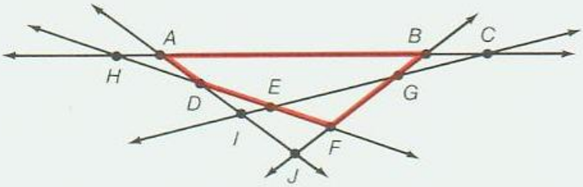


23. Suppose you try to make a square picture frame. You cut four pieces of wood and glue them together so that $AB = BC = CD = AD$. Which of the following statements are known to be true using definitions only?

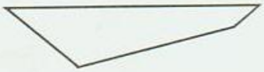
a. *ABCD* is a square.
b. *ABCD* is a rectangle.
c. *ABCD* is a rhombus.
d. *ABCD* is a parallelogram.

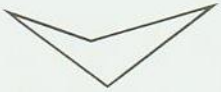
PROBLEM SOLVING

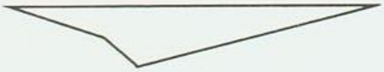
Shown below is a quadrilateral formed by an array of five lines.



These three quadrilaterals can also be found. Name them.



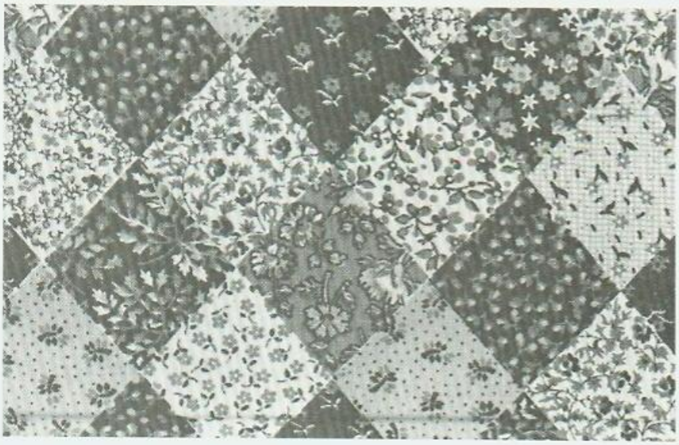




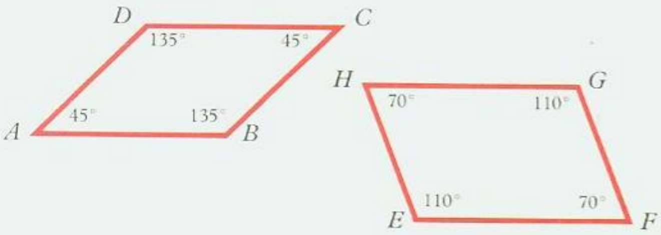
There are at least six more quadrilaterals in the array. Trace and name them.

8–2 Parallelograms

The parallelograms shown in the design have special side and angle relationships.

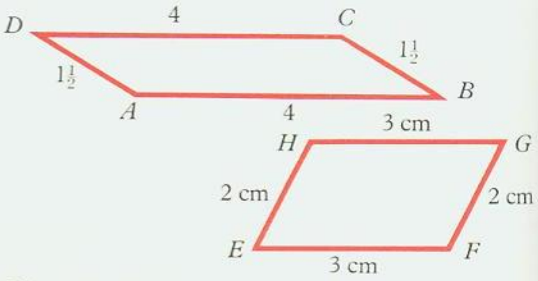


In the parallelograms below the measures of some pairs of opposite angles and opposite sides are given.



Observe that

$\angle A \cong \angle C$, $\angle B \cong \angle D$,
$\angle E \cong \angle G$, $\angle F \cong \angle H$.



Observe that

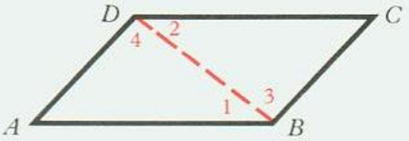
$\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$
$\overline{EF} \cong \overline{GH}$, $\overline{EH} \cong \overline{FG}$.

These observations suggest two basic properties of parallelograms.

Theorem 8–1 The opposite angles of a parallelogram are congruent.

Theorem 8–2 The opposite sides of a parallelogram are congruent.

PROOF



Given: $ABCD$ is a parallelogram.
Prove: $\angle A \cong \angle C$, $\angle B \cong \angle D$
$\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$

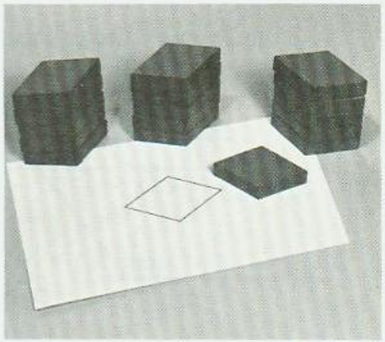
Plan: Draw diagonal $\overline{BD}$ and prove $\triangle ABD \cong \triangle CDB$.

Statements	Reasons
1. $ABCD$ is a parallelogram.	**1.** Given
2. $\overline{AB} \parallel \overline{CD}$	**2.** Definition of parallelogram
3. $\overline{BC} \parallel \overline{AD}$	**3.** Why?
4. $\angle 1 \cong \angle 2$	**4.** If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
5. $\angle 3 \cong \angle 4$	**5.** Why?
6. $\overline{BD} = \overline{BD}$	**6.** Why?
7. $\triangle ABD \cong \triangle CDB$	**7.** Why?
8. $\overline{AB} \cong \overline{CD}$	**8.** CPCTC
9. $\angle A \cong \angle C$	**9.** CPCTC

By repeating this proof using the diagonal $\overline{AC}$ we can prove that $\overline{AD} \cong \overline{BC}$ and $\angle B \cong \angle D$.

APPLICATION

Any production process must include a check on the quality of the item produced. In the production of pattern blocks the check might involve measuring opposite angles of the quadrilateral. The contrapositive of Theorem 8-1 says that if opposite angles are not congruent, then the block cannot be a parallelogram and should be discarded.



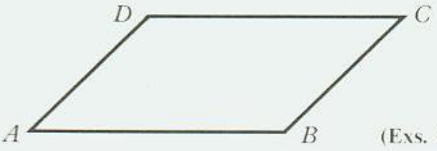
Another important theorem that will be proved as an exercise is stated next.

Theorem 8-3 Each pair of adjacent angles of a parallelogram are supplementary angles.

EXERCISES

A.

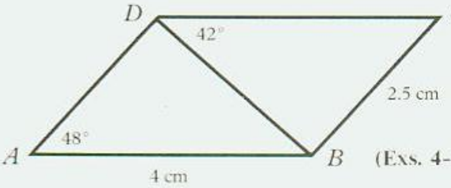
In exercises 1–3, assume *ABCD* is a parallelogram.



(Exs. 1-3)

1. Name two pairs of congruent segments.
2. Name two pairs of congruent angles.
3. Name four pairs of supplementary angles.

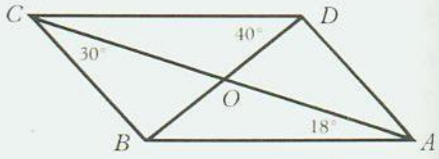
In exercises 4–11, assume *ABCD* is a parallelogram.



(Exs. 4-11)

4. $m\angle C = \underline{?}$
5. $m\angle ABC = \underline{?}$
6. $m\angle ABD = \underline{?}$
7. $m\angle ADB = \underline{?}$
8. $m\angle DBC = \underline{?}$
9. $m\angle ADC = \underline{?}$
10. $AD = \underline{?}$
11. $CD = \underline{?}$

In exercises 12–15, assume *ABCD* is a parallelogram.

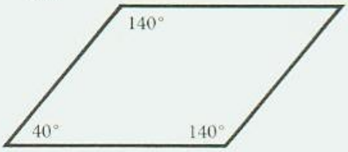


(Exs. 12-15)

12. $m\angle ABC = \underline{?}$
13. $m\angle DOC = \underline{?}$
14. $m\angle ADC = \underline{?}$
15. $m\angle BOC = \underline{?}$
16. Write Theorem 8–1 in if-then form, and then state its contrapositive.
17. Write Theorem 8–2 in if-then form, and then state its contrapositive.

Use the contrapositives in exercises 16 and 17 to determine which of the figures in exercises 18–23 *cannot* be parallelograms.

18.



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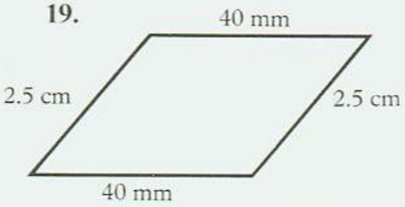

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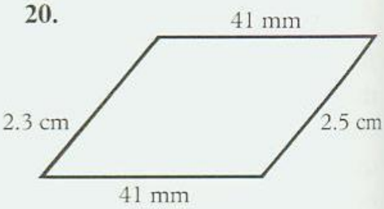

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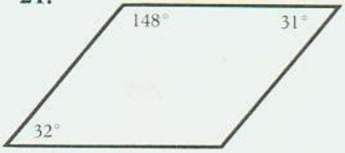

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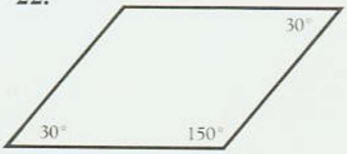

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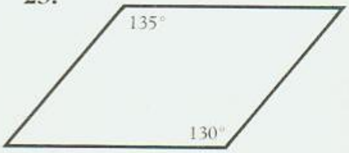
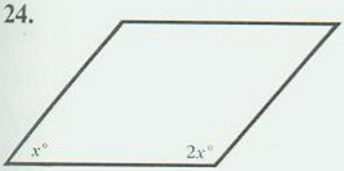


B.

For exercises 24–26, find the measures of all angles of the parallelogram.

24.



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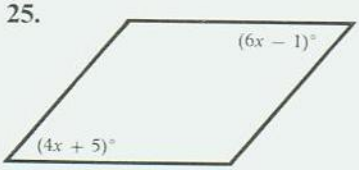

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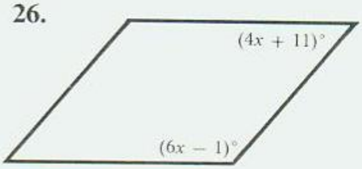
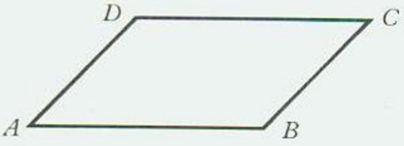


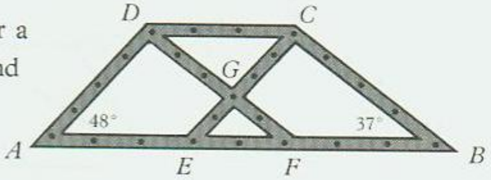
27. $ABCD$ is a parallelogram. If $AB = x + 5$ and $CD = 2x - 7$, find the length of $\overline{AB}$.

28. $ABCD$ is a parallelogram. If $AB = 2x$, $CD = 3y + 4$, $BC = x + 7$, and $AD = 2y$, find the lengths of the sides of the parallelogram.

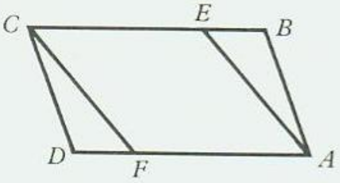


(Exs. 27, 28)

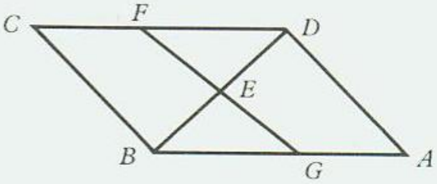
29. Part of the structural support system for a bridge is shown. $\overline{AB} \parallel \overline{CD}$, $\overline{DF} \parallel \overline{CB}$, and $\overline{AD} \parallel \overline{EC}$. Find $m\angle CGF$.



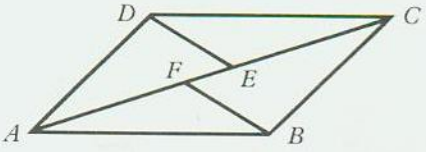
30. Given: $ABCD$ is a parallelogram.
$AECF$ is a parallelogram.
Prove: $\triangle CDF \cong \triangle ABE$



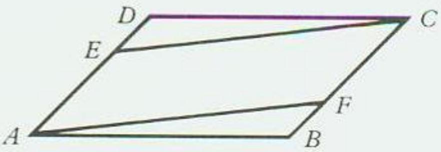
31. Given: $ABCD$ is a parallelogram.
$\overline{FG}$ bisects $\overline{DB}$.
Prove: $\overline{DB}$ bisects $\overline{FG}$.



32. Given: $ABCD$ is a parallelogram.
A, F, E, and C are collinear.
$\overline{AF} \cong \overline{CE}$
Prove: $\overline{DE} \parallel \overline{BF}$



33. Given: $ABCD$ is a parallelogram.
$\overline{AE} \cong \overline{CF}$
Prove: $\overline{CE} \cong \overline{AF}$

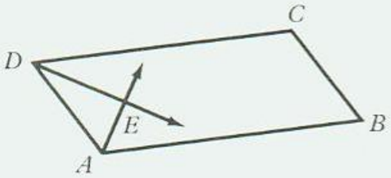


34. Prove that the diagonals of a parallelogram bisect each other.

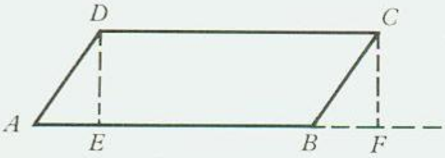
C.

35. Prove that a parallelogram with one right angle is a rectangle.

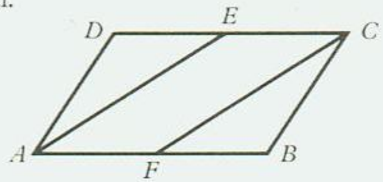
36. **Given:** $ABCD$ is a parallelogram.
$\overrightarrow{AE}$ bisects $\angle A$.
$\overrightarrow{DE}$ bisects $\angle D$.
Prove: $\overline{AE} \perp \overline{DE}$



37. **Given:** $ABCD$ is a parallelogram.
$\overline{DE} \perp \overline{AB}$
$\overline{CF} \perp \overleftrightarrow{AB}$
Prove: $\triangle ADE \cong \triangle BCF$ and $CDEF$ is a rectangle.



38. **Given:** $ABCD$ is a parallelogram.
$\overline{AE}$ bisects $\angle A$.
$\overline{CF}$ bisects $\angle C$.
Prove: $\overline{AE} \cong \overline{CF}$



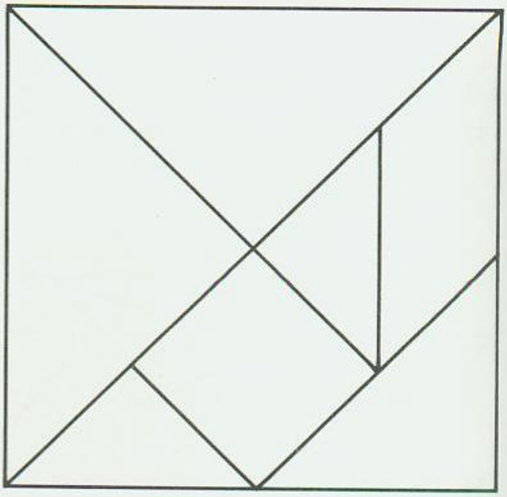
39. Prove Theorem 8-3.

Activity

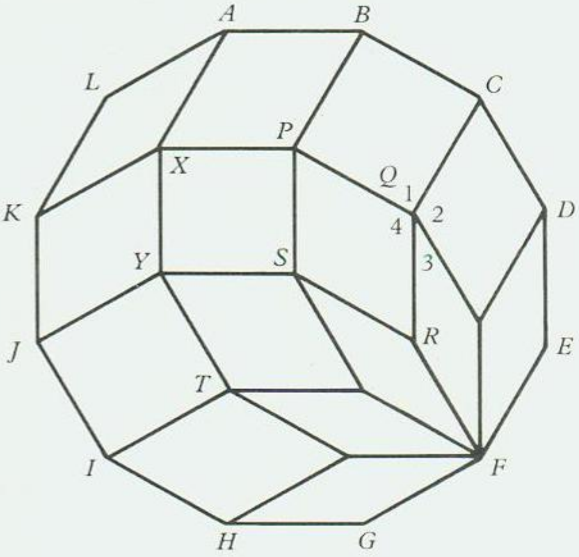
Trace this tangram puzzle and cut out the pieces.

Use the five small pieces to form a square. Can you place the two large pieces around the square to form:

1. a triangle?
2. a parallelogram?
3. a trapezoid?
4. a rectangle?



In the figure a regular 12-gon has been divided into parallelograms. Use the definition of regular polygon and the theorems from this section to answer exercises 40–48. Also accept as given for exercises 40–48 that $m\angle ALK = 150$.



40. $m\angle BPX = \underline{\ ?\ }$

41. $m\angle 1 = \underline{\ ?\ }$

42. $m\angle 2 = \underline{\ ?\ }$

43. $m\angle 3 = \underline{\ ?\ }$

44. $m\angle 4 = \underline{\ ?\ }$

45. Prove that $\overline{AB} \cong \overline{XY}$.

46. Prove that three of the polygons in the figure are squares.

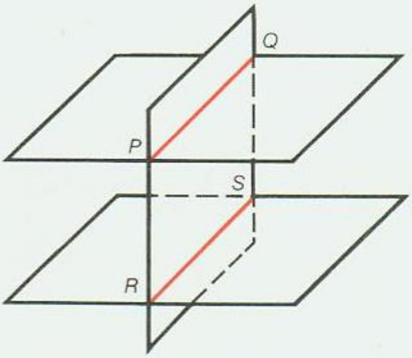
47. Prove that $PQRS$ is a rhombus.

48. Select any parallelogram in the figure. Prove that it is a rhombus.

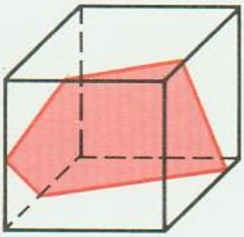
PROBLEM SOLVING

Fact: When two parallel planes in space are cut by a third plane, the intersecting lines are parallel lines. This fact can be used to solve the following problems.

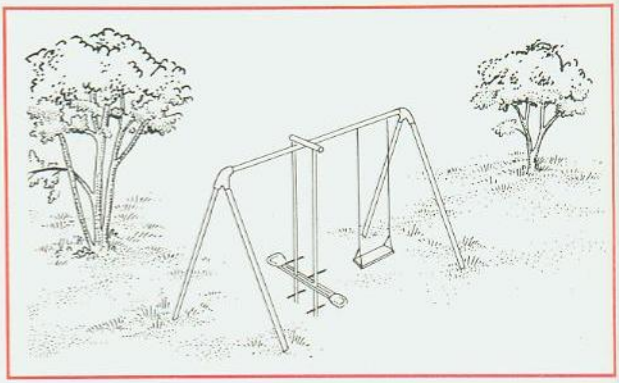
The region common to a cube solid and a plane is called a *cross section* of a cube solid.



1. Why does *each* quadrilateral cross section of a cube solid have at least one pair of parallel edges?
2. Why does *each* pentagonal cross section of a cube solid have two pairs of parallel edges?

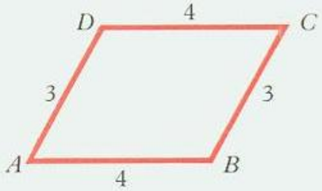


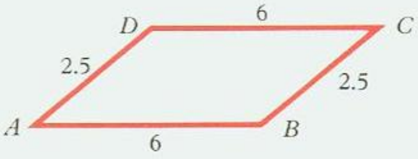
8–3 Quadrilaterals That Are Parallelograms

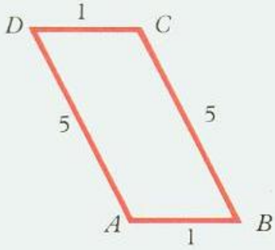


When two children are swinging on a two-person swing, are the seats of the swing always parallel to the top frame? The theorem in this lesson will give the answer.

Consider the quadrilaterals below.



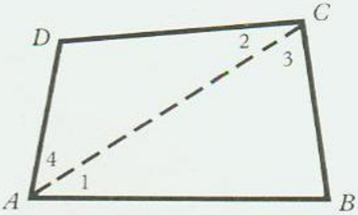




In each case, is *ABCD* a parallelogram?

Theorem 8–4 If opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

PROOF



Given: Quadrilateral *ABCD* with $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{CD}$

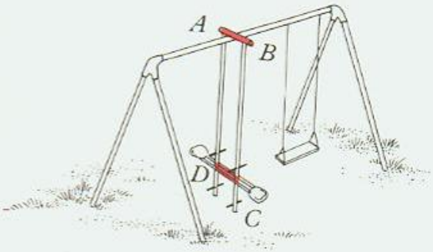
Prove: *ABCD* is a parallelogram.

Plan: Draw auxiliary segment $\overline{AC}$ and prove $\triangle ABC \cong \triangle CDA$.

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\overline{BC} \cong \overline{DA}$	2. Given
3. $\overline{AC} \cong \overline{AC}$	3. Reflexive Property
4. $\triangle ABC \cong \triangle CDA$	4. SSS Postulate
5. $\angle 1 \cong \angle 2$	5. Why?
6. $\overline{AB} \parallel \overline{CD}$	6. Why?
7. $\angle 3 \cong \angle 4$	7. CPCTC
8. $\overline{AD} \parallel \overline{BC}$	8. Why?
9. $ABCD$ is a parallogram.	9. Definition of parallelogram

APPLICATION

The answer to the question posed at the beginning of the lesson is yes. The seats are always parallel to the top frame segment AB. There are four metal poles bolted together at points A, B, C, and D so that $\overline{AB}$ and $\overline{CD}$, and $\overline{AD}$ and $\overline{BC}$ are the same length. Theorem 8–4 tells us that as the swing moves, $ABCD$ is always a parallelogram and $\overline{CD}$ is parallel to $\overline{AB}$.



The next two theorems give other methods for proving that a quadrilateral is a parallelogram.

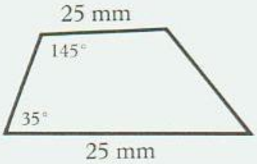
Theorem 8–5 If a quadrilateral has one pair of opposite sides parallel and congruent, then it is a parallelogram.

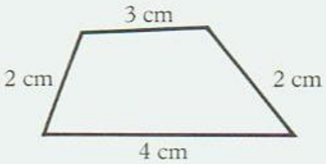
Theorem 8–6 If the opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

EXERCISES

A.

Use the theorems of this section to decide whether each quadrilateral is a parallelogram. Base your decision on the given measurements rather than the shape of the drawings.

1.


2.


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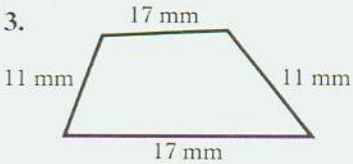

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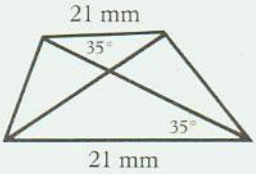

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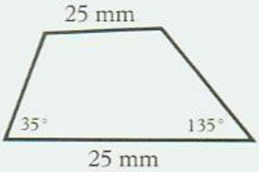

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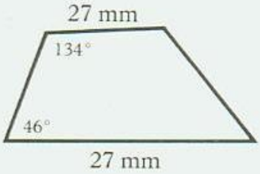
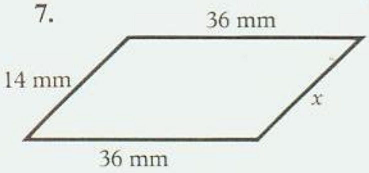
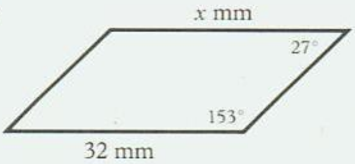


In exercises 7–9, find the value of x that makes the quadrilateral a parallelogram.

7.


8.


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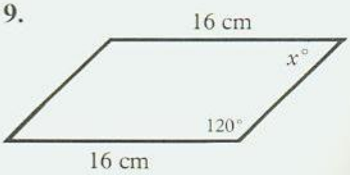


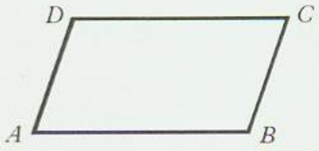
B.

In exercises 10–12, find values of x and y that make $ABCD$ a parallelogram.

10. $AB = 2x + 4$, $CD = 4x - 20$, $AD = 2y$, $BC = y + 5$

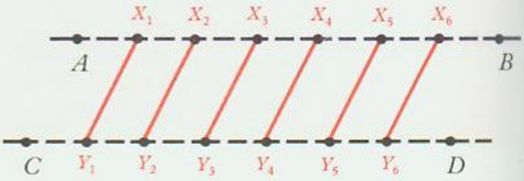
11. $m\angle A = 2x - 60$, $m\angle D = x - 5$, $AB = 4y + 6$, $CD = 6y - 10$

12. $AB = 6x + 30$, $BC = 2x - 5$, $CD = 2y - 10$, $AD = y - 35$



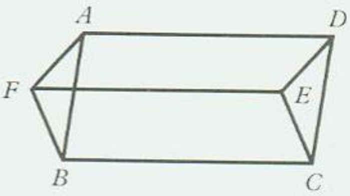
(Exs. 10-12)

13. A parking lot is to be marked for slant parking. A string is stretched from A to B with marks made every 9 feet at X_1, $X_2, \ldots, X_6$. A second string is stretched parallel to $\overline{AB}$ from C to D with marks located every 9 feet at Y_1, $Y_2, \ldots, Y_6$. Why are all painted lines parallel?

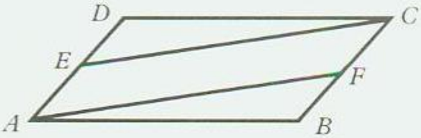


14. Complete the proof of Theorem 8–5.

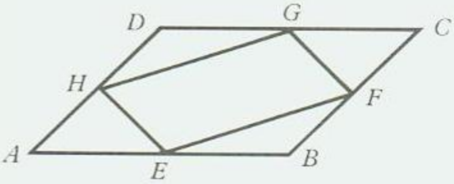
15. **Given:** *ABCD* is a parallelogram.
BCEF is a parallelogram.
Prove: *ADEF* is a parallelogram.



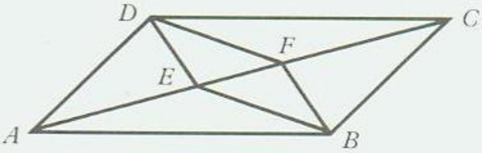
16. **Given:** *ABCD* is a parallelogram.
E is the midpoint of $\overline{AD}$.
F is the midpoint of $\overline{BC}$.
Prove: *AFCE* is a parallelogram.



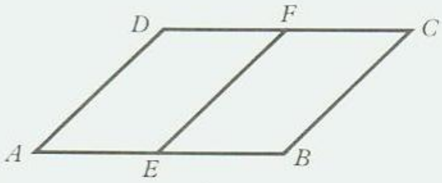
17. **Given:** *ABCD* is a parallelogram.
E, F, G, and *H* are midpoints of sides as shown.
Prove: *EFGH* is a parallelogram.



18. **Given:** *ABCD* is a parallelogram and $AE = CF$.
Prove: *BFDE* is a parallelogram.



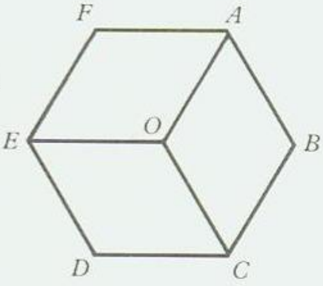
19. **Given:** *ABCD* is a parallelogram.
E is the midpoint of $\overline{AB}$.
F is the midpoint of $\overline{CD}$.
Prove: *AEFD* is a parallelogram.



C.

20. a. **Given:** *ABCDEF* is a regular hexagon.
$m\angle AFE = 120$, $\overline{OA}$, $\overline{OC}$, and $\overline{OE}$ bisect $\angle A$, $\angle C$, and $\angle E$ respectively.
Prove: *ABCO* is a rhombus.

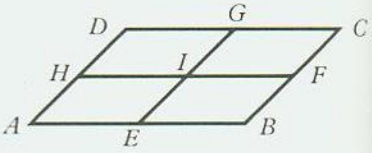
b. Are *CDEO* and *EFAO* also rhombuses?



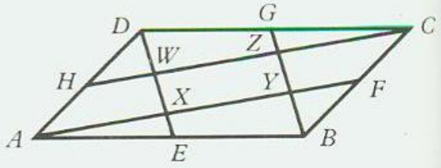
21. Prove that if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

22. Prove that if angles B and D in quadrilateral $ABCD$ are both supplementary to angle A, then $ABCD$ is a parallelogram.

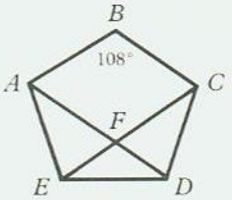
23. **Given:** $ABCD$ is a parallelogram.
E, F, G, and H are midpoints of sides.
Prove: The figure $AEIH$ is a parallelogram.



24. **Given:** $ABCD$ is a parallelogram.
E, F, G, and H are midpoints of sides as shown.
Prove: The figure $WXYZ$ is a parallelogram.



25. **Given:** $ABCDE$ is a regular pentagon.
Prove: The figure $ABCF$ is a rhombus.

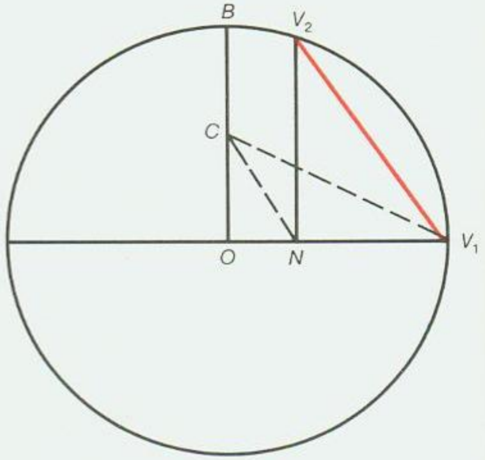


Activity

Draw a large circle with center O and follow the directions below to construct a regular pentagon.

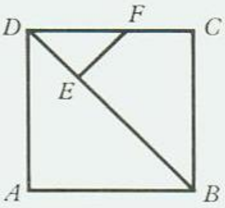
1. Label any point on the circle V_1 and construct $\overline{OB}$ perpendicular to $\overline{OV_1}$.
2. Join V_1 to C, the midpoint of $\overline{OB}$.
3. Bisect angle OCV_1 to obtain the point N on $\overline{OV_1}$.
4. Construct the perpendicular to $\overline{OV_1}$ at N and obtain the point V_2.

The segment $\overline{V_1V_2}$ is one side of a regular pentagon.

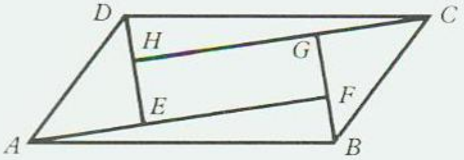


26. Prove that if the diagonals of a parallelogram are congruent, then the figure is a rectangle.

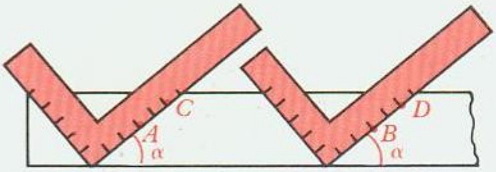
27. Given: $ABCD$ is a square.
$BE = BC$, $\overline{EF} \perp \overline{BD}$
Prove: $DE = EF = FC$



28. Given: $EFGH$ is a parallelogram.
$\overline{HD} \cong \overline{FB}$, $\overline{AE} \cong \overline{CG}$
Prove: $ABCD$ is a parallelogram.



29. A carpenter wants to construct parallel lines on a board. This can be done by using a carpenter square twice. Each time the tool is placed at the same angle with the board and equal units are marked off. Explain why this method assures that $\overline{AB}$ will be parallel to $\overline{CD}$.



PROBLEM SOLVING

When a plane intersects a cube, the region common to the plane and the cube solid is called a *cross section* of the cube solid.

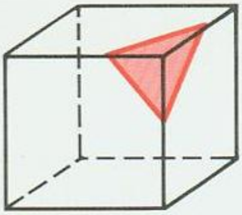
1. How many of these quadrilaterals and their interiors occur as a cross section of a cube?

   a. square **b.** non-square rectangle

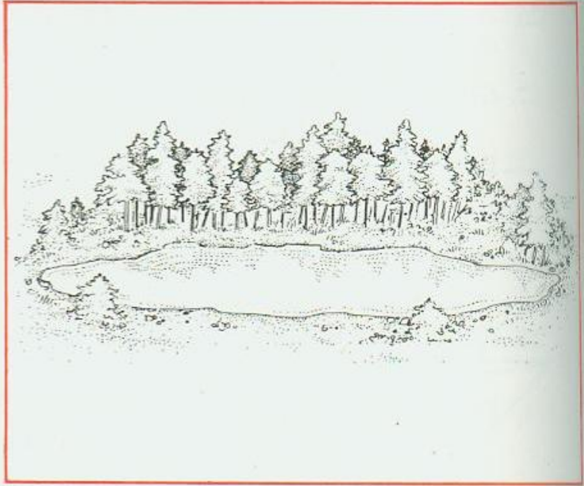
   c. rhombus **d.** non-rhombic parallelogram

   e. trapezoid **f.** kite

2. For each type of cross section that exists trace the cube at the right and sketch the cross section.

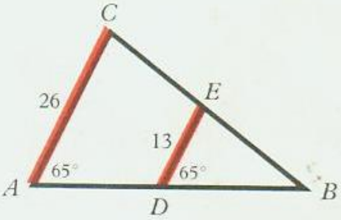


8–4 The Midsegment Theorem

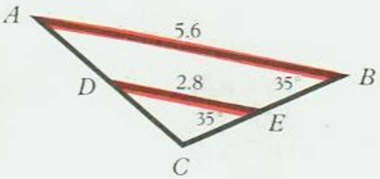


A surveying team needs to find the distance across a large pond. The team chooses any point. From that point they measure to each side of the pond. They locate the two points that are halfway between the edge of the pond and their chosen point. The distance between these two midpoints will be one-half the distance across the pond. The theorem in this lesson helps to explain why.

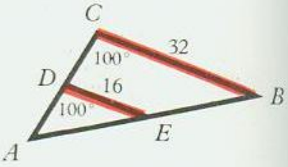
In the triangles below, D and E are midpoints. The segments and angles have the measures shown.



Observe: $DE = \frac{1}{2}AC$.
Since $\angle EDB \cong \angle CAB$,
$\overline{DE} \parallel \overline{AC}$.



Observe: $DE = \frac{1}{2}AB$.
Since $\angle CED \cong \angle CBA$,
$\overline{DE} \parallel \overline{AB}$.



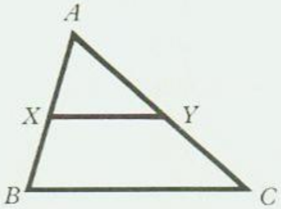
Observe: $DE = \frac{1}{2}CB$.
Since $\angle ADE \cong \angle ACB$,
$\overline{DE} \parallel \overline{CB}$.

Theorem 8–7 **Midsegment Theorem.** A segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

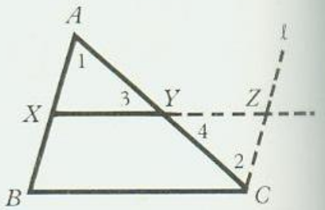
PROOF

Given: Any $\triangle ABC$ with
X the midpoint of $\overline{AB}$,
Y the midpoint of $\overline{AC}$.

Prove: $\overline{XY} \parallel \overline{BC}$ and
$XY = \frac{1}{2}BC$

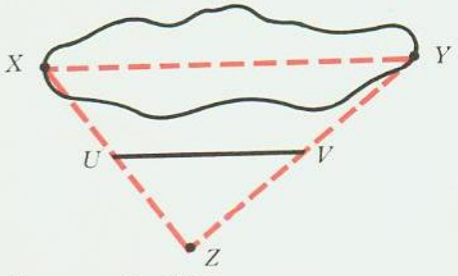


Plan: Draw a line ℓ through C and parallel to $\overline{AB}$. Then extend $\overline{XY}$ until it intersects ℓ at Z. Show that two congruent triangles are formed (Steps 3–6). Then show that $BCZX$ is a parallelogram (Steps 10–13).



Statements	Reasons
1. X is the midpoint of $\overline{AB}$. Y is the midpoint of $\overline{AC}$.	1. Given
2. Line ℓ is drawn through C and parallel to $\overline{AB}$, and $\overline{XY}$ is extended to form $\triangle CYZ$.	2. Construction
3. $AY = YC$	3. Definition of midpoint
4. $\angle 1 \cong \angle 2$	4. If two lines are parallel, then alternate interior angles are congruent.
5. $\angle 3 \cong \angle 4$	5. Why?
6. $\triangle AXY \cong \triangle CZY$	6. ASA Postulate
7. $XY = ZY$	7. CPCTC
8. Y is the midpoint of $\overline{XZ}$.	8. Definition of midpoint
9. $XY = \frac{1}{2}XZ$	9. Algebra
10. $CZ = AX$	10. Statement 6 and CPCTC
11. $AX = XB$	11. Definition of midpoint
12. $CZ = XB$; $\overline{CZ} \parallel \overline{AB}$	12. Transitive property; Statement 2
13. $BCZX$ is a parallogram.	13. If a quadrilateral has one pair of opposite sides parallel and congruent, then it is a parallelogram.
14. $\overleftrightarrow{XY} \parallel \overleftrightarrow{BC}$	14. Definition of parallelogram
15. $\overline{XZ} \cong \overline{BC}$	15. Opposite sides of a parallelogram are congruent.
16. $XY = \frac{1}{2}BC$	16. Substitution in Statements 9 and 15

Note that the conclusion of the theorem follows from statements 14 and 16.



APPLICATION

The surveyor's method was a direct application of Theorem 8-7. Since U and V are the midpoints of $\overline{ZX}$ and $\overline{ZY}$, $UV = \frac{1}{2}XY$, or $XY = 2UV$.

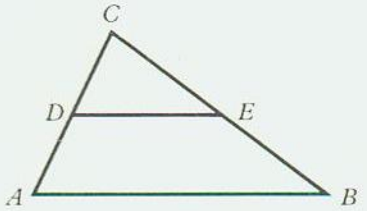
The following theorem can be proved using Theorem 8-7. See exercise 33.

Theorem 8–8 The midpoints of the sides of a quadrilateral are the vertices of a parallelogram.

EXERCISES

A.

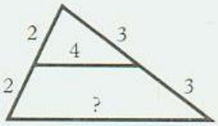
In this figure D and E are midpoints.



1. If $AB = 8$, then $DE = \underline{?}$
2. If $AC = 9$, then $AD = \underline{?}$
3. If $BE = 5$, then $BC = \underline{?}$
4. If $AB = 15$, then $DE = \underline{?}$
5. If $DE = 17$, then $AB = \underline{?}$

For exercises 6–14, state either the missing number(s) or state "cannot determine."

6.



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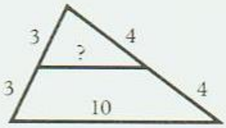
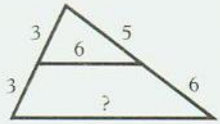


8.



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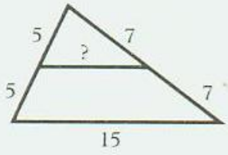

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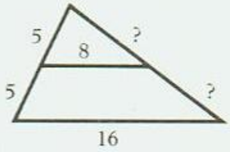

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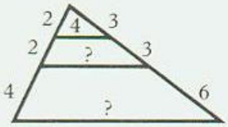

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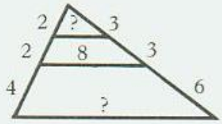

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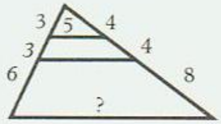

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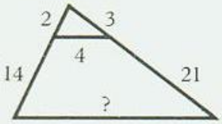
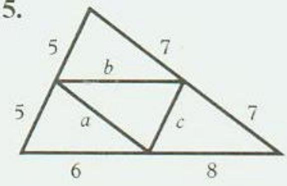


In exercises 15–17, exactly one of the numbers a, b, or c can be determined. Find it.

15.



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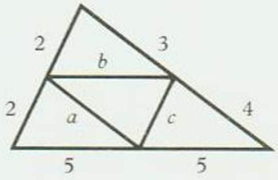

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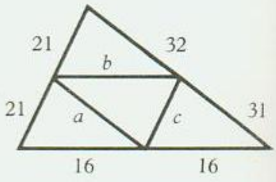
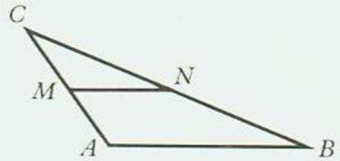


B.

In the triangle on the right M and N are midpoints.



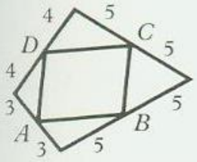
(Exs. 18–19)

18. If $MN = x + 8$ and $AB = 4x + 14$, find the lengths MN and AB.

19. If $AM = x + 5$, $MC = 2y + 6$, $MN = 2x - 5$, and $AB = y + 8$, find the lengths MN and AB.

For exercises 20–22, use Theorem 8-8 to decide whether or not $ABCD$ is a parallelogram.

20.



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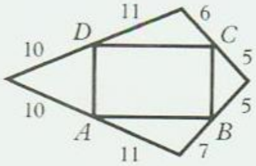

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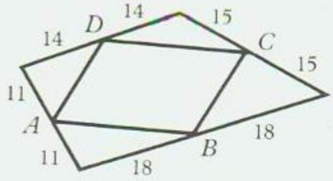
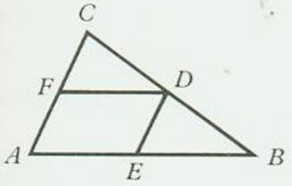
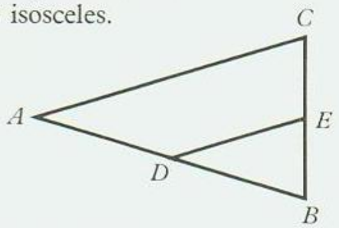


For exercises 23–26 write complete two-column proofs.

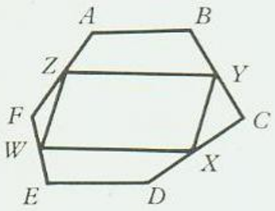
23. **Given:** F is the midpoint of $\overline{AC}$.
D is the midpoint of $\overline{BC}$.
E is the midpoint of $\overline{AB}$.
Prove: $AEDF$ is a parallelogram.



24. **Given:** $\triangle ABC$ is isosceles with $AB = AC$.
D is the midpoint of $\overline{AB}$.
E is the midpoint of $\overline{CB}$.
Prove: $\triangle BDE$ is isosceles.

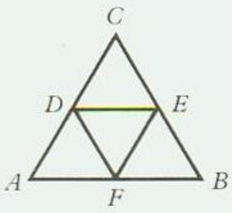


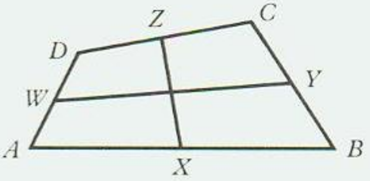
25. **Given:** $ABCDEF$ is a hexagon with $\overline{AB} \parallel \overline{DE}$ and $AB = DE$.
W, X, Y, and Z are midpoints of sides as shown.
Prove: $WXYZ$ is a parallelogram.



(*Hint:* Use auxiliary segments $\overline{AE}$ and $\overline{BD}$.)

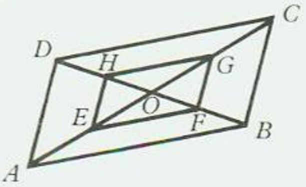
26. **Given:** $\triangle ABC$ is equilateral.
D, E, and F are midpoints of sides as shown.
Prove: $\triangle DEF$ is equilateral.





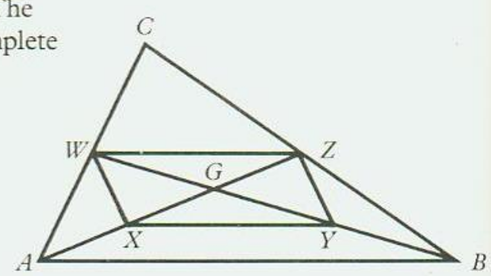
27. **Given:** W, X, Y, and Z are midpoints of the sides of quadrilateral $ABCD$.
Prove: $\overline{WY}$ and $\overline{XZ}$ bisect each other.
(*Hint:* Use auxiliary lines.)

28. **Given:** $ABCD$ is a parallelogram.
E, F, G, and H are midpoints of $\overline{AO}$, $\overline{BO}$, $\overline{CO}$, and $\overline{DO}$, respectively.
Prove: $EFGH$ is a parallelogram.



C.

Exercises 29–32 all have the following given information. The statement in any one of these exercises may be used to complete any succeeding exercise.



Given: $\overline{AZ}$ and $\overline{BW}$ are medians of $\triangle ABC$.
X is the midpoint of $\overline{AG}$.
Y is the midpoint of $\overline{BG}$.

29. Prove that $\overline{WZ} \parallel \overline{XY}$.

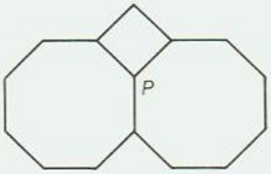
30. Prove that $WXYZ$ is a parallelogram.

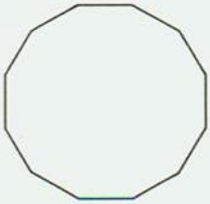
31. Prove that $AX = XG = GZ$ and $BY = YG = GW$.

32. Prove that the centroid of a triangle trisects each median. That is, the centroid divides the median into one third of the median and two thirds of the median. (The centroid is the point of intersection of the medians.)

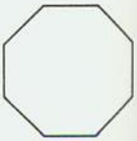
Activity

You can fit three of these polygons around a point P like this.





Dodecagon



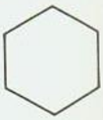
Octagon

Use tracing paper to show as many different ways as you can to

1. fit three polygons around a point.
2. fit four polygons around a point.
3. fit five polygons around a point.

Square

Equilateral triangle



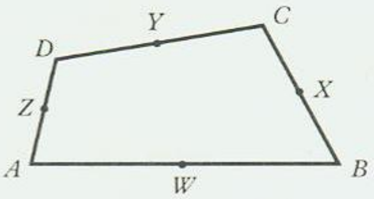
Hexagon

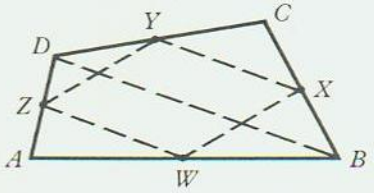
Can you draw a design by continuing to fit polygons together?

33. Complete the following proof of Theorem 8–8.

Given: $ABCD$ is any quadrilateral.
W, X, Y, and Z are midpoints of sides of $ABCD$ as shown.
Prove: $WXYZ$ is a parallelogram.

Plan: Add auxiliary segments $\overline{DB}$, $\overline{ZW}$, $\overline{YX}$, $\overline{ZY}$, and $\overline{WX}$. Apply Theorem 8–7 to $\triangle ABD$ and $\triangle BCD$.





Exercises 34–36 have the following given information.

Given:

a. $ABCD$ is a parallelogram.

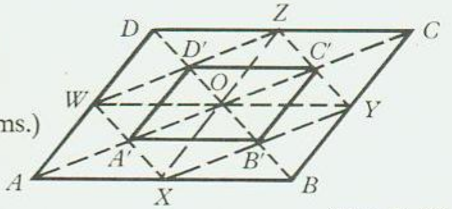
b. W, X, Y, and Z are the midpoints of the sides.

c. Each dashed line is a diagonal of a polygon.

34. Prove that $WOZD$ is a parallelogram. A similar proof will show that $AXOW$, $XBYO$, and $OYCZ$ are parallelograms. (*Hint:* First show that $AXZD$ and $WYCD$ are parallelograms.)

35. Prove that $A'B'C'D'$ is a parallelogram.

36. Prove that each side of $A'B'C'D'$ is parallel to and half as long as the corresponding side of $ABCD$.

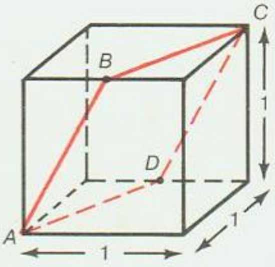


(Exs. 34–36)

PROBLEM SOLVING

Pictured here is a cube, all of whose edges have length 1. Suppose that points B and D are the midpoints of the edges shown.

1. Show that $ABCD$ is a rhombus.
2. Find the lengths of the sides.
3. Find the lengths of the diagonals $\overline{BD}$ and $\overline{AC}$.

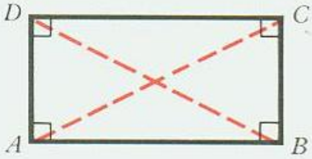


8–5 Rectangles, Rhombuses, and Squares

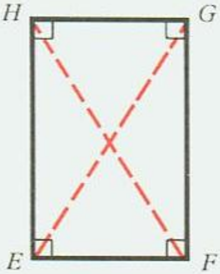
Recall from the definitions of rectangle, rhombus, and square that they are all special types of parallelograms. These figures occur in a variety of industrial settings. Frequently it is necessary to check that an object actually is one of these special parallelograms. For example, a contractor must make sure that the foundation wall of a building is an exact rectangle.

In this lesson we shall study how these three types of parallelograms are determined by their diagonals.

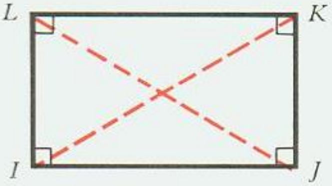
These parallelograms are also rectangles.



Observe: $\overline{AC} \cong \overline{BD}$.



Observe: $\overline{EG} \cong \overline{FH}$.



Observe: $\overline{IK} \cong \overline{JL}$.

These observations support this theorem.

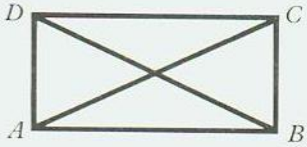
Theorem 8–9 A parallelogram is a rectangle if and only if its diagonals are congruent.

We must prove two things.

I. If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

II. If a parallelogram is a rectangle, then the diagonals are congruent.

Outline for I.

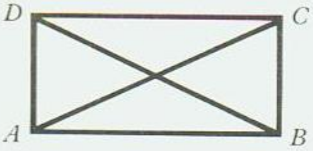


Given: $ABCD$ is a parallelogram.
$\overline{AC} \cong \overline{BD}$

Prove: $ABCD$ is a rectangle.

Plan: Prove that $\triangle ABD \cong \triangle BAC$, and that $\angle A$ and $\angle B$ are congruent and supplementary. Similarly for $\angle C$ and $\angle D$.

Outline for II.



Given: $ABCD$ is a rectangle.

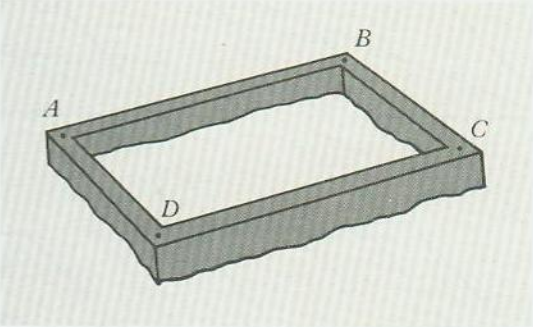
Prove: $\overline{AC} \cong \overline{BD}$.

Plan: Prove that $\triangle ABD \cong \triangle BAC$.

This proof will be completed as an exercise.

APPLICATION

The concrete footings for a house form a rectangular shape slightly larger than the rectangle of the house. On these footings the contractor must locate four points A, B, C, and D that are to become the actual corners of the house. These four points must be accurately located so that $ABCD$ is a perfect rectangle. After measuring to make $AB = CD$ and $AD = BC$, the next step is to measure the diagonals. If $AC = BD$, then $ABCD$ is a rectangle.



Is $ABCD$ an exact rectangle?

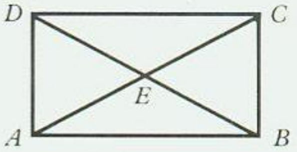
Proofs of these last two theorems will be found as exercises.

Theorem 8–10 A parallelogram is a rhombus if and only if its diagonals are perpendicular to each other.

Theorem 8–11 A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

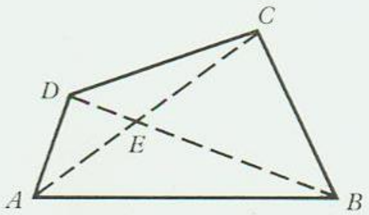
EXERCISES

A.



1. In rectangle *ABCD* name all pairs of congruent segments.

For exercises 2–4, assume that quadrilateral *ABCD* has been distorted into the desired shape.

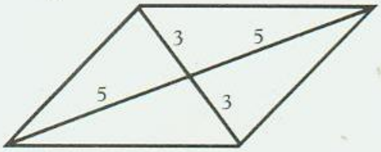


(Exs. 2-4)

2. If *ABCD* were a parallelogram, name all pairs of congruent segments.
3. If *ABCD* were a rhombus, name all the angles that must be right angles.
4. If *ABCD* were a rhombus, name all the angles that must be congruent to ∠*CAB*.

Which of these parallelograms would be rectangles? (Assume that the given information is correct even though the figure may be distorted.)

5.



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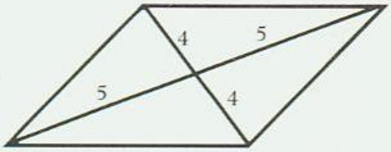

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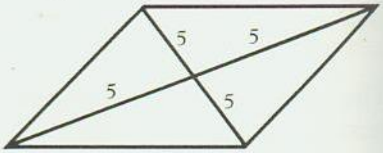

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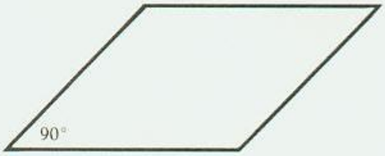

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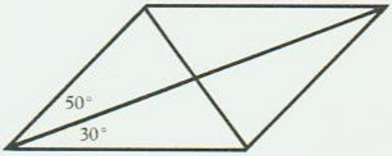

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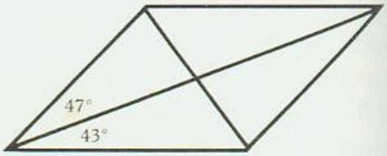
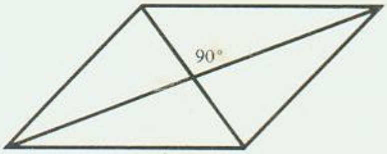


Which of these parallelograms would be rhombuses? (Assume that the given information is correct even though the figure may be distorted.)

11.



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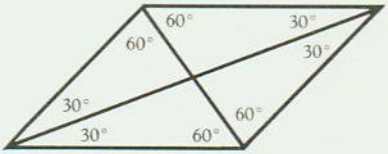

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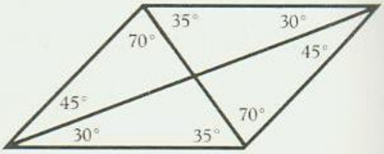


For exercises 14–20, decide whether the given statement is true or false.

14. Every rectangle is a parallelogram.

15. Every rhombus is a rectangle.

16. Every square is a rhombus.

17. Some rhombuses are squares.

18. Some rhombuses are rectangles that are not squares.

19. If the diagonals of a quadrilateral are congruent, then the figure is a rectangle.

20. The diagonals of a square are perpendicular.

B.

For exercises 21–25 draw, if possible, a parallelogram satisfying each of the following conditions. If it is not possible, write "not possible."

21. All angles congruent

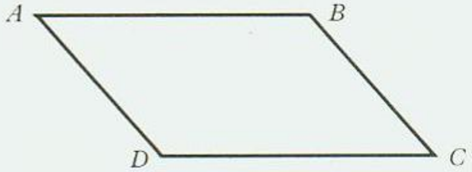
22. Diagonals that bisect each other

23. All sides congruent with diagonals that are not perpendicular

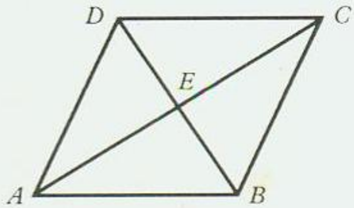
24. No right angles with congruent diagonals

25. Congruent and perpendicular diagonals

26. $ABCD$ is a parallelogram.
$AB = 2x + 4$
$DC = 3x - 11$
$AD = x + 19$
Show that $ABCD$ is a rhombus.



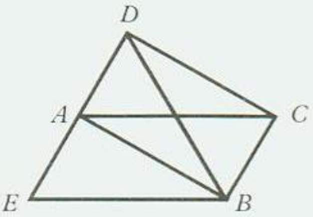
27. $ABCD$ is a rhombus.
$m\angle DEC = 4x + 10$
$m\angle DAB = 3x + 4$
Find $m\angle ABC$.



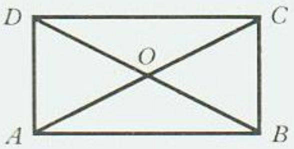
(Ex. 27)

28. $ABCD$ is a parallelogram.
$AB = 4x - 5$ $\quad AC = 3x - 2$
$CD = 2x + 23$ $\quad BD = 2x + 12$
Show that $ABCD$ is a rectangle.

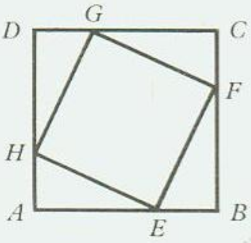
29. **Given:** $ABCD$ is a rectangle.
$ACBE$ is a parallelogram.
Prove: $\triangle DBE$ is isosceles.



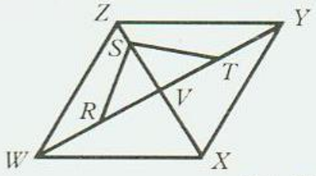
30. **Given:** $ABCD$ is a rectangle.
Prove: $\triangle ABO$ is isosceles.



31. **Given:** $ABCD$ is a square.
$AH = DG = CF = BE$
Prove: $EFGH$ is a square.



32. **Given:** $WXYZ$ is a rhombus.
R is the midpoint of $\overline{WV}$.
T is the midpoint of $\overline{VY}$.
S is a point on $\overline{VZ}$.
Prove: $\triangle RST$ is isosceles.

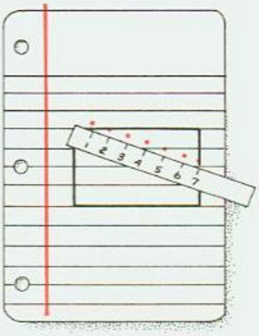


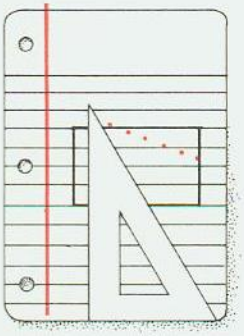
(Exs. 32, 33)

33. **Given:** $WXYZ$ is a rhombus.
Prove: $\overline{WY}$ and $\overline{XZ}$ divide $WXYZ$ into four congruent triangles.

34. Complete the proof of Theorem 8-9.

Activity





Suppose you need to draw a 7-column chart for a social studies report. Your space is 5 inches wide. Here is how you can draw the chart without doing any calculations.

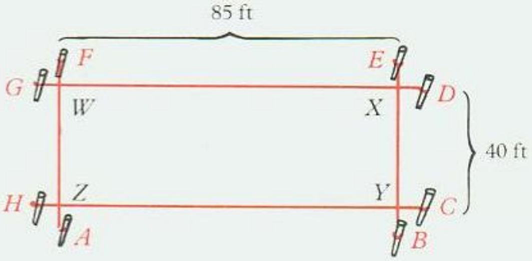
1. Set a ruler as shown and mark the seven inch-points.
2. Draw vertical lines through each point parallel to the sides of the chart (or perpendicular to the bottom of the chart).

Experiment with drawing 9 columns in a 7-inch space, 4 columns in a 5-inch space.

35. A building is to be $85' \times 40'$. Stakes are placed as shown and string is stretched. The outside corners of the building will be at the points where the strings cross.

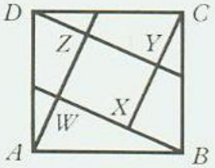
a. After the string has been stretched the contractor measures $\overline{WY}$ and $\overline{XZ}$. Why?

b. If WY is 93 feet and XZ is 94 feet, which way should stakes E and F be moved to make $WXYZ$ a rectangle?

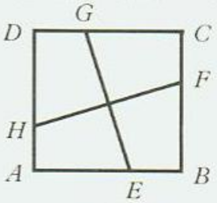


C.

36. Given: $WXYZ$ is a square.
$AW = BX = CY = DZ$
Prove: $ABCD$ is a square.



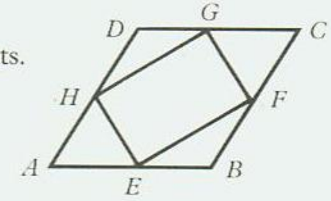
37. Given: $ABCD$ is a square.
$AH = DG = CF = BE$
Prove: $EG = HF$ and $\overline{EG} \perp \overline{HF}$



38. Prove Theorem 8–10.

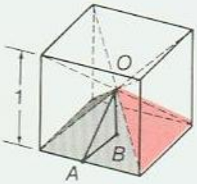
39. Prove Theorem 8–11.

40. Given: $ABCD$ is a rhombus.
E, F, G, and H are midpoints.
Prove: $EFGH$ is a rectangle.



PROBLEM SOLVING

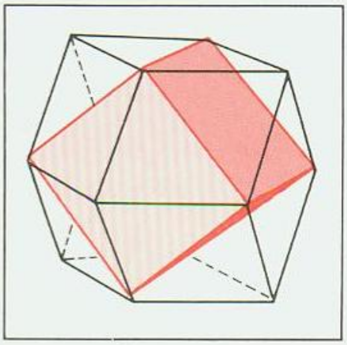
If the center O of a cube is joined to the vertices of a face, a square-based pyramid is formed. Suppose the length of an edge of the cube is 1.



1. If B is the center of the square base, what is the length of $\overline{OB}$?
2. If A is the midpoint of an edge, what is the length of $\overline{OA}$?

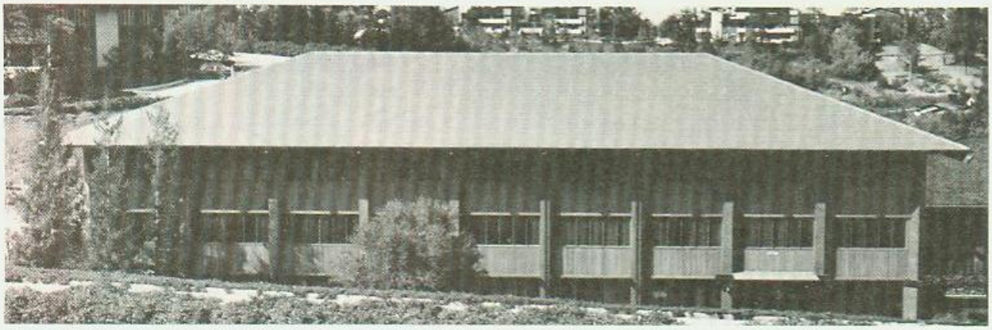
If square-based pyramids like the one above are affixed on the faces of a cube, a solid is formed whose faces are rhombuses.

3. How many rhombic faces does this solid have?



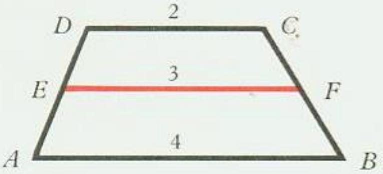
Cube with square-based pyramid affixed to each face.

8–6 Trapezoids



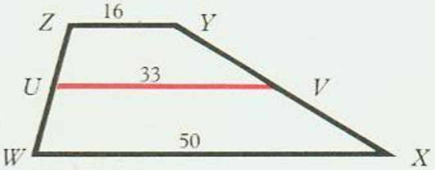
Recall that a trapezoid is a quadrilateral with exactly one pair of parallel sides. One example of a trapezoid would be the roof of a house. In this lesson we shall study a theorem about trapezoids that could be useful in estimating the construction costs of a project.

E and F are midpoints as shown.



Observe that $EF = \frac{1}{2}(AB + CD)$ and $\overline{EF} \parallel \overline{AB} \parallel \overline{CD}$.

U and V are midpoints as shown.



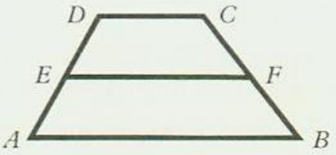
Observe that $UV = \frac{1}{2}(WX + YZ)$ and $\overline{UV} \parallel \overline{WX} \parallel \overline{YZ}$.

Theorem 8–12 The segment joining the midpoints of the two nonparallel sides of a trapezoid is parallel to the two bases and has a length equal to one half the sum of the lengths of the bases.

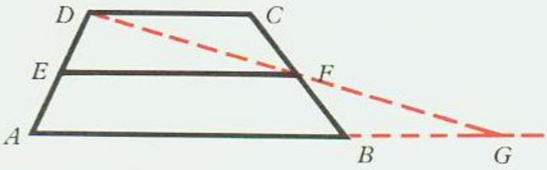
PROOF

Given: $ABCD$ is a trapezoid with $\overline{DC} \parallel \overline{AB}$.
E the midpoint of $\overline{AD}$ and
F the midpoint of $\overline{BC}$.

Prove: $\overline{EF} \parallel \overline{AB}$, $\overline{EF} \parallel \overline{DC}$,
and $EF = \frac{1}{2}(AB + CD)$



Plan: Extend $\overline{AB}$ and $\overline{DF}$ to meet at G. Then prove that F is the midpoint of $\overline{DG}$ and use the Midsegment Theorem.

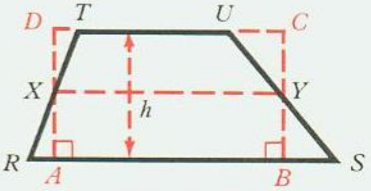


Statements	Reasons
1. Extend $\overline{AB}$.	**1.** Construction
2. Draw $\overrightarrow{DF}$ intersecting $\overrightarrow{AB}$ at G.	**2.** Construction
3. $\overline{DC} \parallel \overline{AB}$	**3.** Definition of trapezoid
4. $\angle BGF \cong \angle CDF$	**4.** Why?
5. $\overline{CF} \cong \overline{BF}$	**5.** Why?
6. $\angle BFG \cong \angle DFC$	**6.** Why?
7. $\triangle BFG \cong \triangle CFD$	**7.** Why?
8. $\overline{DF} \cong \overline{GF}$	**8.** Why?
9. F is the midpoint of $\overline{DG}$.	**9.** Why?
10. $\overline{EF} \parallel \overline{AB}$ and $\overline{EF} \parallel \overline{DC}$	**10.** Midsegment Theorem

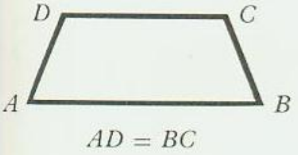
It remains to show that $EF = \frac{1}{2}(AB + CD)$. This proof will be completed in an exercise.

APPLICATION

In estimating the construction costs of a roof the area of the trapezoid must be calculated. This area is equal to the area of the rectangle $ABCD$. By Theorem 8–12, $XY = \frac{1}{2}(RS + TU)$, and area $(ABCD) = h(XY) = \frac{1}{2}h(RS + TU)$.



The following theorem states properties of a special kind of trapezoid.



$\overline{AD}$ and $\overline{BC}$ are nonparallel sides. $\angle A$ and $\angle B$ together are called *base angles.* $\angle C$ and $\angle D$ together are another pair of base angles.

Definition 8–6

An **isosceles trapezoid** is a trapezoid with congruent nonparallel sides.

The proof of this theorem is left for the student in exercises 13 and 15.

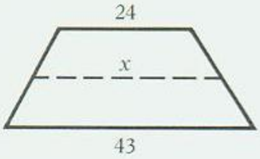
Theorem 8–13 In an isosceles trapezoid base angles are congruent and the diagonals are congruent.

EXERCISES

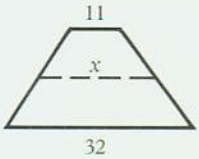
A.

For exercises 1–9 the dashed line joins the midpoints of the two nonparallel sides of the trapezoid. Find the value of x.

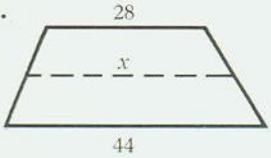
1.



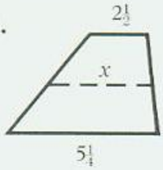
2.



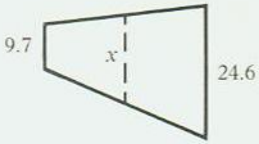
3.



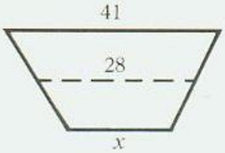
4.



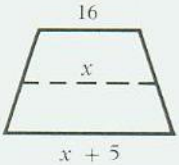
5.



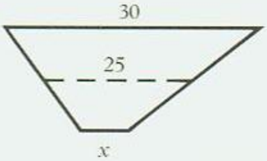
6.



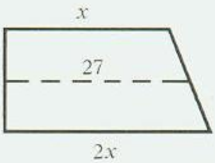
7.



8.

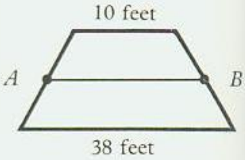


9.



B.

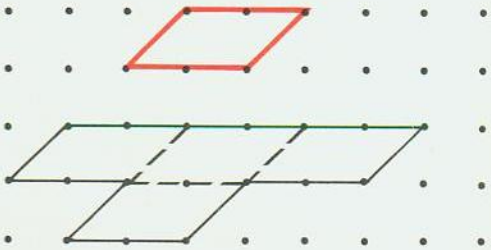
10. A dam is constructed with a trapezoidal cross section 10 feet across the top and 38 feet across the base. What is the "average width" AB of the dam?



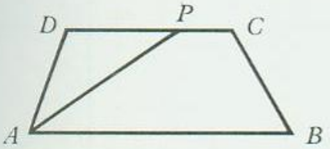
Activity

Shown here is a polygon formed by placing together along common edges four congruent copies of the given parallelogram.

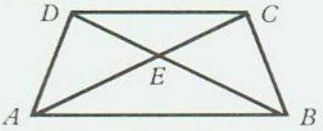
Draw at least nine different shaped polygons using this method of placing together four copies of the given parallelogram.



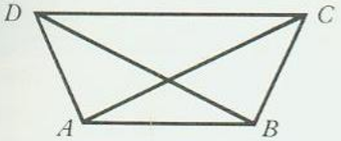
11. **Given:** $ABCD$ is a trapezoid with $\overline{AB} \parallel \overline{CD}$. P is on $\overline{CD}$ so that $\overline{AP}$ bisects $\angle A$.
Prove: $\triangle APD$ is isosceles.



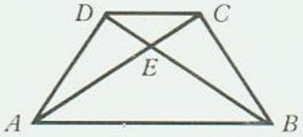
12. **Given:** $ABCD$ is a trapezoid with $\overline{AB} \parallel \overline{CD}$. $\overline{AD} \cong \overline{BC}$ $\overline{AC}$ and $\overline{BD}$ intersect at E.
Prove: $\triangle CDE$ is isosceles.



13. **Given:** $ABCD$ is an isosceles trapezoid with $\overline{AB} \parallel \overline{CD}$.
Prove: $\overline{AC} \cong \overline{BD}$

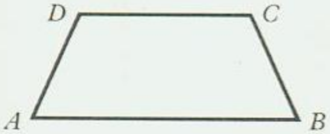


14. **Given:** $ABCD$ is a trapezoid with $\overline{AB} \parallel \overline{CD}$ and $\overline{AE} \cong \overline{BE}$.
Prove: $\overline{AD} \cong \overline{BC}$

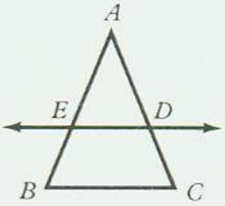


C.

15. **Given:** $ABCD$ is an isosceles trapezoid with $\overline{AB} \parallel \overline{CD}$.
Prove: $\angle A \cong \angle B$
(*Hint:* Construct a line through D parallel to $\overline{BC}$.)



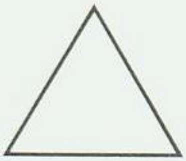
16. **Given:** $\triangle ABC$ is isosceles with $\overline{AB} \cong \overline{AC}$, $\angle AED \cong \angle B$.
Prove: $BCDE$ is a trapezoid with $\overline{BE} \cong \overline{CD}$.



PROBLEM SOLVING

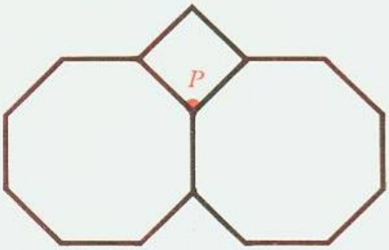
1. Trace a regular hexagon. Can you cut it to form
   - **a.** 6 equilateral triangles?
   - **b.** 2 isosceles trapezoids?
   - **c.** 3 congruent rhombuses?

2. Trace two regular hexagons. Can you cut them to produce equilateral triangles?
3. Trace an equilateral triangle. Can you cut it to form 3 congruent 5-sided figures.
   (*Hint:* Use a figure Y at the center.)



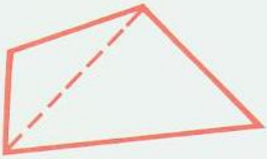
8–7 The Angles of a Polygon

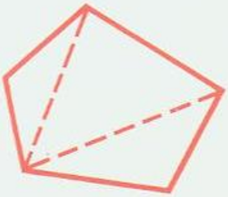
The activity on page 280 asks you to look for combinations of regular polygons that fit together around a point P.

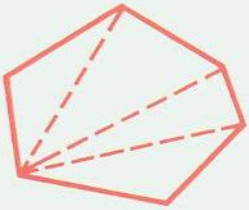


The measures of the vertex angles of polygons determine whether or not they really fit together.

We first ask what is the sum of the angle measures of a polygon? To answer this question, we draw diagonals from one vertex of the polygon to form triangles.


Quadrilateral


Pentagon


Hexagon

In each of these cases the sum of the measures of the angles of the polygon is the sum of the measures of the angles of the triangles. This observation leads to this table.

Polygon	Number of sides	Number of triangles	Sum of the measures of the angles
quadrilateral	4	2	$2(180^\circ) = 360^\circ$
pentagon	5	3	$3(180^\circ) = 540^\circ$
hexagon	6	4	$4(180^\circ) = 720^\circ$
⋮	⋮	⋮	⋮
n-gon	n	$n - 2$	$(n - 2)180^\circ$

The inductive reasoning demonstrated by the preceding table suggests these two theorems.

Theorem 8-14 The sum of the measures of the angles of a convex polygon of n sides is $(n - 2)180^\circ$.

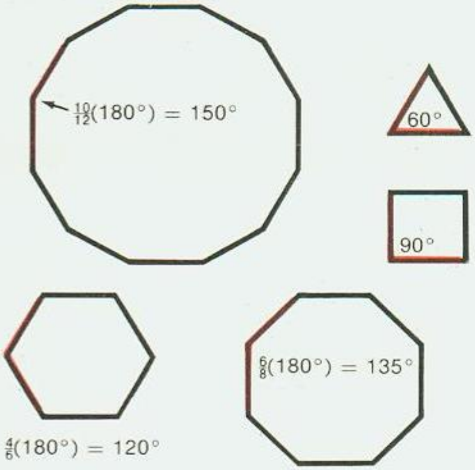
Theorem 8-15 The measure of an angle of a regular polygon of n sides is $\frac{(n-2)}{n}180^\circ$.

APPLICATION

An artist working on a mosaic may ask which combinations of three or four of these regular polygons will fit together around a point as shown at the beginning of the lesson.

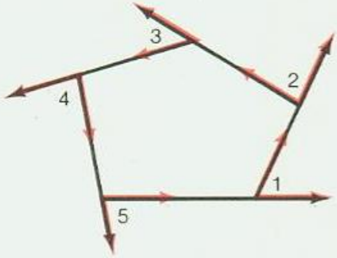
Step 1 Use Theorem 8-15 to find the measure of the angles of each of these regular polygons.

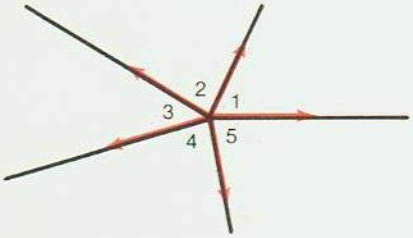
Step 2 Use trial and error to find combinations of the numbers found in Step 1 whose sum is 360°.



$\frac{10}{12}(180^\circ) = 150^\circ$
60°
90°
$\frac{6}{8}(180^\circ) = 135^\circ$
$\frac{4}{6}(180^\circ) = 120^\circ$

Consider the pentagon on the left. An exterior angle at each vertex is labeled. If we cut out these exterior angles and arrange them around a point, we see that their sum is 360°.





$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360$

Theorem 8-16 The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360°.

EXERCISES

A.

In exercises 1–3 the number of sides of a convex polygon is given. The diagonals from one vertex of the polygon divides the polygon into how many triangles?

1. 10
2. 25
3. x

In exercises 4–9 the number of sides of a convex polygon is given. Find the sum of the measures of the angles of the polygon.

4. 6
5. 12
6. 24
7. 36
8. 100
9. p

In exercises 10–15 the sum of the measures of the interior angles is given. Find the number of sides of the polygon.

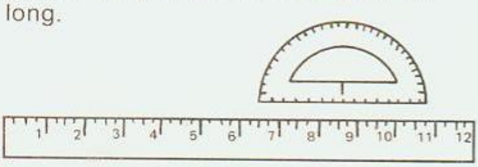
10. 7020°
11. 1980°
12. 6120°
13. 1800°
14. 1260°
15. 3420°

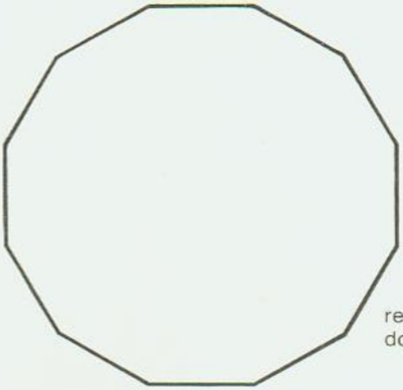
In exercises 16–21 the number of sides of a regular polygon is given. Find the measure of a vertex angle of the polygon.

16. 7
17. 9
18. 10
19. 15
20. 20
21. 100

Activity

1. Use a ruler and protractor to draw a regular 12-gon with each edge 3 cm long.
2. Use a ruler and protractor to draw a regular 15-gon with each edge 3 cm long.





regular dodecagon

B.

22. The sum of the measures of seven angles of an octagon is 1000°. What is the measure of the eighth angle?

23. What is the measure of each exterior angle of a regular octagon? Of a regular 12-gon?

24. How many sides does a regular polygon have if each exterior angle has a measure of 15°? of 18°?

25. How many sides does a regular polygon have if each interior angle has a measure of 108°? of 144°?

26. Show that two regular pentagons and a regular 10-gon will fit together around a point. (See application.)

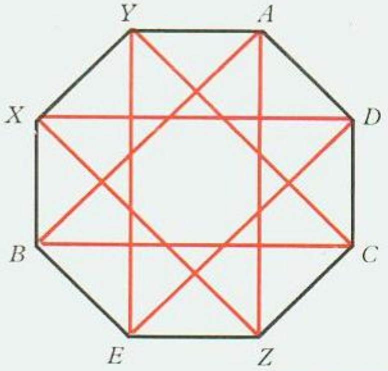
27. Show that an equilateral triangle, a regular 7-gon, and a regular 42-gon will fit together around a point.

C.

28. Find the number of sides of a polygon if the sum of the measures of its interior angles is twice the sum of the measures of its exterior angles.

29. Inscribed in a regular octagon is a star polygon. Find $m\angle ABC$. Prove that your answer is correct.

30. Prove that $\overline{AB} \parallel \overline{DE}$.



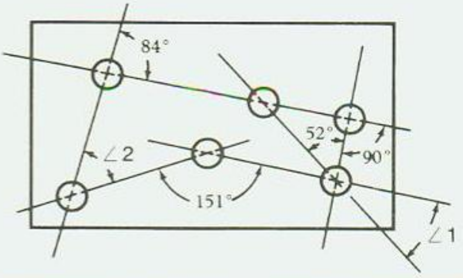
(Exs. 29–30)

PROBLEM SOLVING

A steel plate is to be made with holes drilled as shown.

1. If $m\angle 1 = 37$, find $m\angle 2$.

2. If $m\angle 1 = 43$, find $m\angle 2$.



Important Ideas—Chapter 8

Terms

Trapezoid (p. 261)
Parallelogram (p. 261)
Rectangle (p. 261)
Rhombus (p. 261)
Square (p. 261)
Isosceles trapezoid (p. 289)

Theorems

8-1 The opposite angles of a parallelogram are congruent.

8-2 The opposite sides of a parallelogram are congruent.

8-3 Each pair of adjacent angles of a parallelogram are supplementary angles.

8-4 If the opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

8-5 If a quadrilateral has one pair of opposite sides parallel and congruent, then it is a parallelogram.

8-6 If the opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

8-7 **Midsegment Theorem.** A segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

8-8 The midpoints of the sides of a quadrilateral are the vertices of a parallelogram.

8-9 A parallelogram is a rectangle if and only if its diagonals are congruent.

8-10 A parallelogram is a rhombus if and only if its diagonals are perpendicular to each other.

8-11 A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

8-12 The segment joining the midpoints of the two nonparallel sides of a trapezoid is parallel to the two bases and has a length equal to one half the sum of the lengths of the bases.

8-13 In an isosceles trapezoid base angles are congruent and the diagonals are congruent.

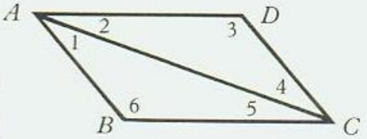
8-14 The sum of the measures of the angles of a convex polygon of n sides is $(n-2)180^\circ$.

8-15 The measure of an angle of a regular polygon of n sides is $\frac{(n-2)}{n}180^\circ$.

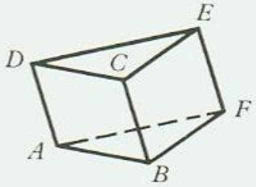
8-16 The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360°.

Chapter 8—Review

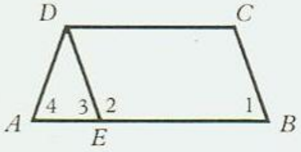
1. Indicate on your paper whether the following statements are true or false.
   - **a.** If the diagonals of a parallelogram are congruent, then the figure must be a rectangle.
   - **b.** If the diagonals of a parallelogram are perpendicular, then the figure must be a square.
   - **c.** If a figure is a square, then it must be a rhombus.
   - **d.** If the diagonals of a trapezoid are congruent, then the figure is a parallelogram.
2. Suppose $ABCD$ is a parallelogram, $m\angle B = 110$, and $m\angle 2 = 30$. Find $m\angle 4$.



3. **Given:** Figure $ABCD$ is a parallelogram. $\overline{EF} \parallel \overline{DA}$, $\overline{EF} \cong \overline{DA}$
   Prove: Figure $BCEF$ is a parallelogram.

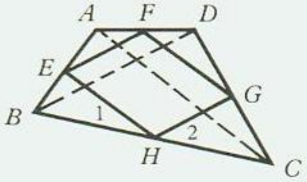


4. **Given:** Figure $BCDE$ is a rhombus. E is the midpoint of $\overline{AB}$, $m\angle 1 = 60$.
   Prove: Figure $ABCD$ is an isosceles trapezoid.
5. **Given:** $EBCD$ is a parallelogram, $m\angle 1 = m\angle 4$.
   Prove: $AD = DE$.



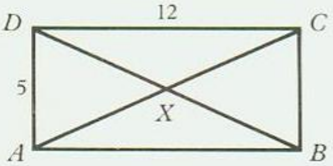
(Exs. 4, 5)

6. Suppose E, F, G, and H are midpoints. If $m\angle 1 = 30$ and $m\angle 2 = 50$, find $m\angle EFG$.
7. Suppose E, F, G, and H are midpoints. If $AC = 12$ and $BD = 8$, find $EF + FG + GH + EH$.



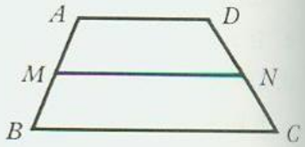
(Exs. 6, 7)

8. Find the measure of each angle of a regular 12-sided polygon.
9. If four angles of a pentagon have measures of 100°, 70°, 150°, and 120°, find the measure of the fifth angle.
10. Suppose $ABCD$ is a rectangle. If $AD = 5$ and $CD = 12$ find BX.



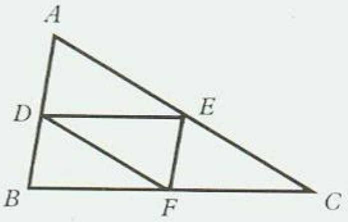
Chapter 8—Test

1. Indicate on your paper whether the following statements are true or false.
   - **a.** If a quadrilateral has one pair of congruent sides, then it is a parallelogram.
   - **b.** If a quadrilateral has congruent diagonals, then it must be a rectangle.
   - **c.** The sum of the exterior angles of a regular pentagon is 360°.
   - **d.** If a parallelogram has one right angle, it is a rectangle.
2. *ABCD* is a trapezoid. *M* and *N* are midpoints. $AD = 6$, and $MN = 10$. Find *BC*.



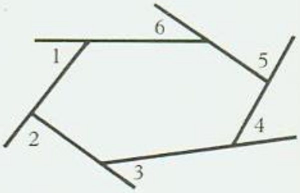
(Ex. 2)

3. Suppose *D*, *E*, and *F* are midpoints. If $AB = 4$, $AC = 5$, and $BC = 6$, find $DE + EF + DF$.
4. **Given:** $\triangle ABC$ is equilateral.
   D, *E*, and *F* are midpoints.
   Prove: $\triangle DEF$ is equilateral.



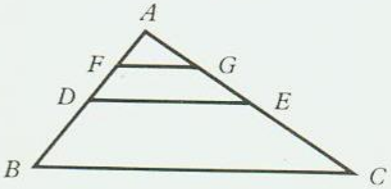
(Exs. 3, 4)

5. *PQRS* is a parallelogram. If $m\angle P = 4x + 20$ and $m\angle Q = x + 10$, find $m\angle P$.
6. If $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 290$, find $m\angle 6$.



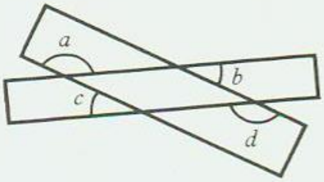
(Ex. 6)

7. If each angle of a regular polygon has a measure of 156°, how many sides does the polygon have?
8. **Given:** $AD = DB$ and $AE = EC$
   $AF = FD$ and $AG = GE$
   Prove: $\overline{FG} \parallel \overline{BC}$



(Ex. 8)

9. The figures shown are two overlapping rectangles. Find the sum, $a + b + c + d$.



Algebra Review

Solve the following proportions for x.

1. $\frac{x}{2} = \frac{6}{3}$
2. $\frac{3}{4} = \frac{6}{x}$
3. $\frac{x}{x-2} = \frac{21}{15}$
4. $\frac{x-3}{6} = \frac{4-x}{-5}$
5. $\frac{5}{6} = \frac{5x}{x+60}$
6. $\frac{x}{3} = \frac{4}{x-1}$
7. $\frac{2x-3}{2} = \frac{5x}{6}$
8. $\frac{x-2}{x+5} = \frac{6}{20}$
9. $\frac{x+2}{5} = \frac{1}{x-2}$

Solve each system.

10. $2x - 4y = 30$
    $3x + 2y = -3$
11. $y = 3x - 5$
    $x = -2y - 10$
12. $2(x-2) + y = -18$
    $3x + 2(y-1) = -26$
13. $2x + 5y = 7$
    $x = -\frac{3}{2}$
14. $5x + 8y = 1$
    $2x - 7y = -20$
15. $2x + y + 3 = 0$
    $8x - \frac{y}{3} + 25 = 0$
16. $x + 2y - 4 = 0$
    $\frac{5}{2}x - 13y - 10 = 0$
17. $\frac{3}{2}p + \frac{4}{3}q = 2$
    $8q = 3p + 4$
18. $m - \frac{n-2}{3} = 0$
    $0.2m + 0.3n = 5$

Solve for x.

19. $\sqrt{x} - 9 = 0$
20. $\sqrt{x+4} = 5$
21. $\sqrt{x+2} + 4 = 7$
22. $\sqrt{x^2 - 9} = 4$
23. $\sqrt{2+x} = 0$
24. $\sqrt{x} + 4 = \sqrt{x} - 2$

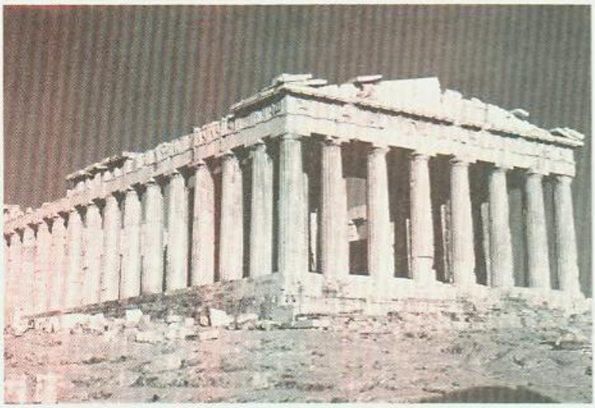
Solve for x.

25. $(x+2)(x-5) = 0$
26. $(2x+5)(3x-1) = 0$
27. $x^2 - 10x + 25 = 0$
28. $(x-4)^2 = 9$
29. $x^2 - x - 12 = 0$
30. $4x^2 - 9 = 0$

Solve.

31. The measures of the angles of a triangle are in a ratio of 2:3:5. Find the measures of the angles.
32. The perimeter of a rectangle is 160. If the ratio of the width to the length is 7:9, find the dimensions of the rectangle.

Architecture: The Golden Rectangle



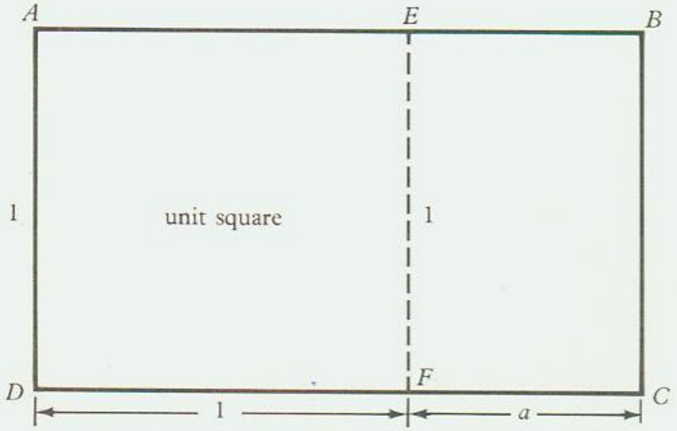
The *Golden Rectangle* was considered by the early Greeks to be one of the most beautifully proportioned geometric forms. It has been used by architects for centuries in planning temples, skyscrapers, and buildings of all kinds.

The Greeks built the Parthenon in Athens in the 5th century B.C. The rectangle that encloses the front face is a Golden Rectangle.

The *Golden Rectangle* is a rectangle such that if a unit square is cut off one end, the sides of the remaining smaller rectangle will be in the same ratio as those in the original rectangle. Since the ratios of pairs of corresponding sides of the large and small rectangle ($ABCD$ and $EBCF$) are equal, we can use this ratio

$$\frac{1 + a}{1} = \frac{1}{a}$$

and calculate the length of the longest side of a Golden Rectangle with width one. Find this length.

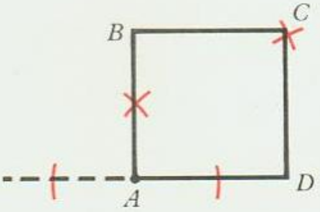


A Golden Rectangle

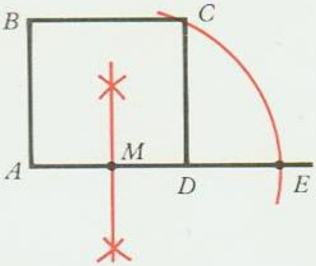
Constructing a Golden Rectangle

Using compass and straightedge, follow the steps below to construct a Golden Rectangle.

Step 1 Construct a unit square $ABCD$.



Step 2 Construct the midpoint of side $\overline{AD}$. Using M as center and radius MC, draw an arc which intersects $\overrightarrow{AD}$ at point E.



Step 3 Construct $\overline{EF} \perp \overline{AE}$ and complete the Golden Rectangle $ABFE$.